Test 2A, Math of Democracy

Dr. Adam Graham-Squire, Fall 2019

Name: Key! 25 min

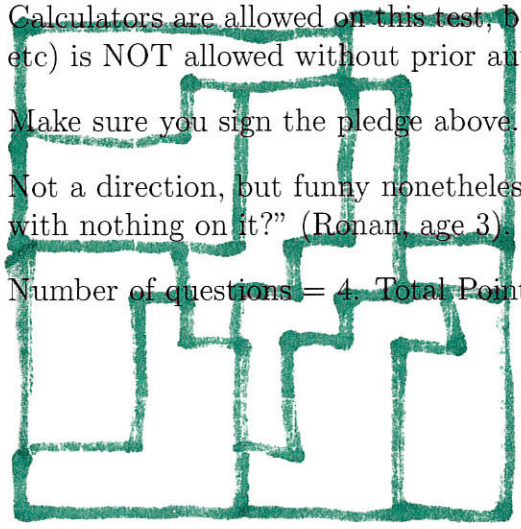
I pledge that I have neither given nor received any unauthorized assistance on this exam.

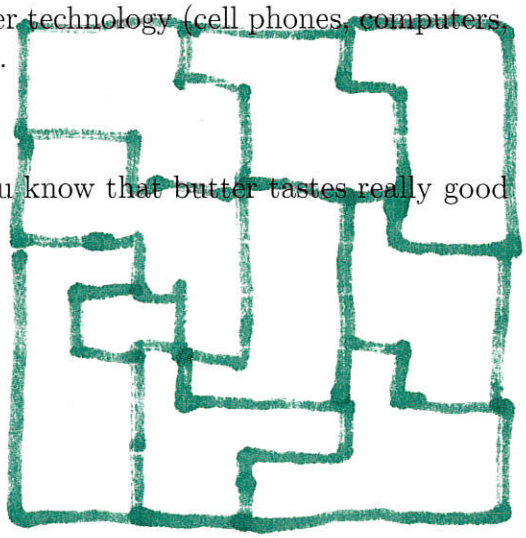
_____

(*signature*)

DIRECTIONS

1. Don't panic.
2. **Show/explain all of your work**. A correct answer with insufficient work will lose points.
3. Read each question carefully, and make sure you answer the the question that is asked. If the question asks for an explanation, make sure you give one.
4. Clearly indicate your answer.
5. Calculators are allowed on this test, but any other technology (cell phones, computers, etc) is NOT allowed without prior authorization.
6. Make sure you sign the pledge above.
7. Not a direction, but funny nonetheless: "Did you know that butter tastes really good with nothing on it?" (Ronan, age 3).
8. Number of questions = 4. Total Points = 20.





1. (6 points) In the 9 by 9 Squaretopia below, there are 35 Filled squares and 46 Dotted squares. The Squaretopia must be divided into 9 districts of 9 squares each. All of the maps below are identical, I just gave you multiple copies in case you want to do scratch work. I also put two more copies on the next page. Do the following:

   (a) Calculate what would be a proportional distribution of districts to each party.

   (b) What is the theoretical maximum number of districts that the (i) Filled and (ii) Dotted party can each win?

   (c) Draw districts that are as favorable as possible to the Filled party. Are your districts as good as you could practically do? Why?

   (d) Draw districts that are as favorable as possible to the Dotted party. Are your districts as good as you could practically do? Why?

Test A → do one → Test B

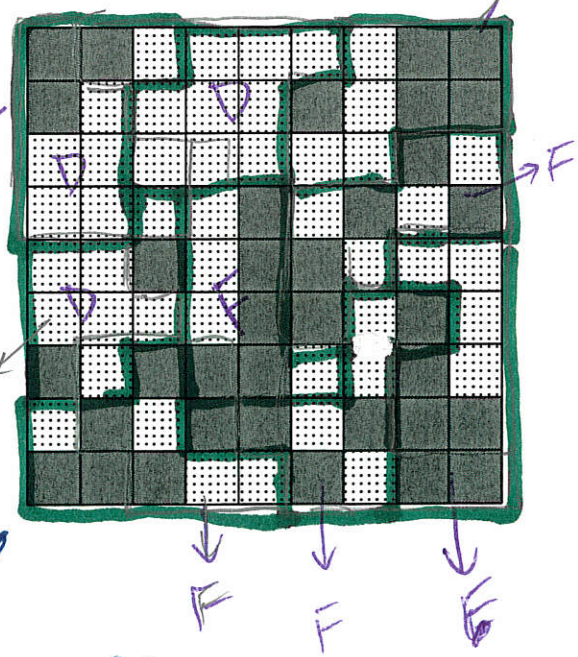
(a) $\frac{35}{81} \times 9 = 3.88 \approx 4$ districts for Filled (or 3 and 6)

$\frac{46}{81} \times 9 = 5.12 \Rightarrow 5$ districts for Dotted

(b) Filled can win up to $\frac{35}{5} = 7$ districts (b/c need 5 blocks to win a ~~dot~~ block district)

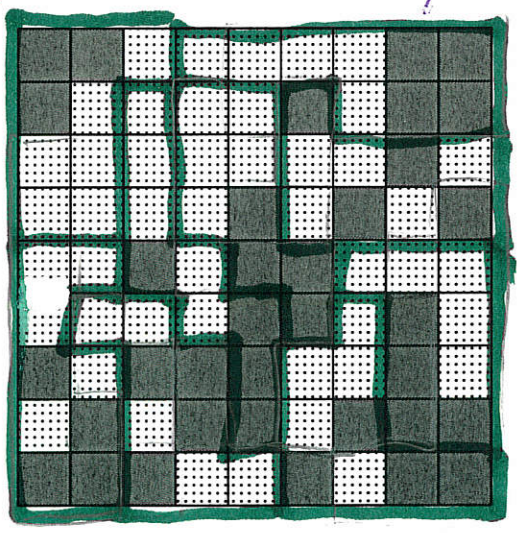
Dotted can win up to $\frac{46}{5} = 9.2 \Rightarrow 9$ (all) districts.

Practically, cannot get more than 6 districts b/c these 3 blocks cannot be united with another filled block



(c)

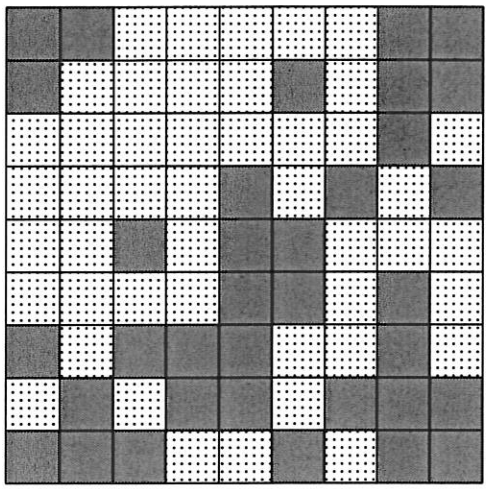
Test A

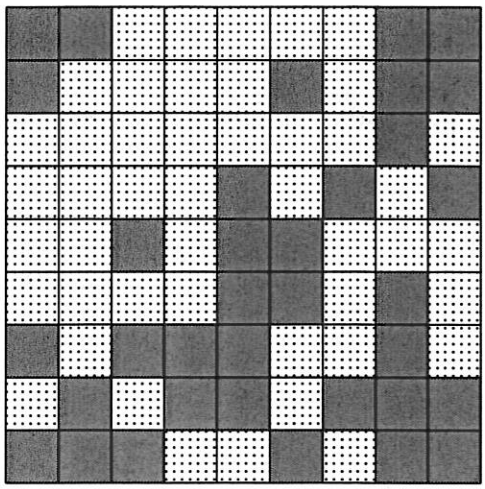


Dots win every district

(d)

Test B





2. (4 points) Calculate the efficiency gap for ~~one of~~ the gerrymandered maps you made in question 1, part (c) ~~or (d)~~. Explain how the efficiency gap score does or does not demonstrate gerrymandering.

(c) 3 Dotted and 6 filled $\Rightarrow$

for filled, $V = \frac{36}{81}$ and $S = \frac{6}{9}$

$\Rightarrow$ E.G. $= 2V - S - \frac{1}{2}$

$2\left(\frac{35}{81}\right) - \frac{6}{9} - \frac{1}{2} = -0.302$ ~~$= -0.278$~~ ✓✓

Threshold is $\frac{2}{9} = 0.22$, so the E. gap ✓

does indicate gerrymandering b/c

$|-0.302| = 0.302$ is greater than the threshhold of 0.22. ✓

Test B: 9 dotted, 0 filled $\Rightarrow 2\left(\frac{46}{81}\right) - \frac{9}{9} - \frac{1}{2} = -0.364$

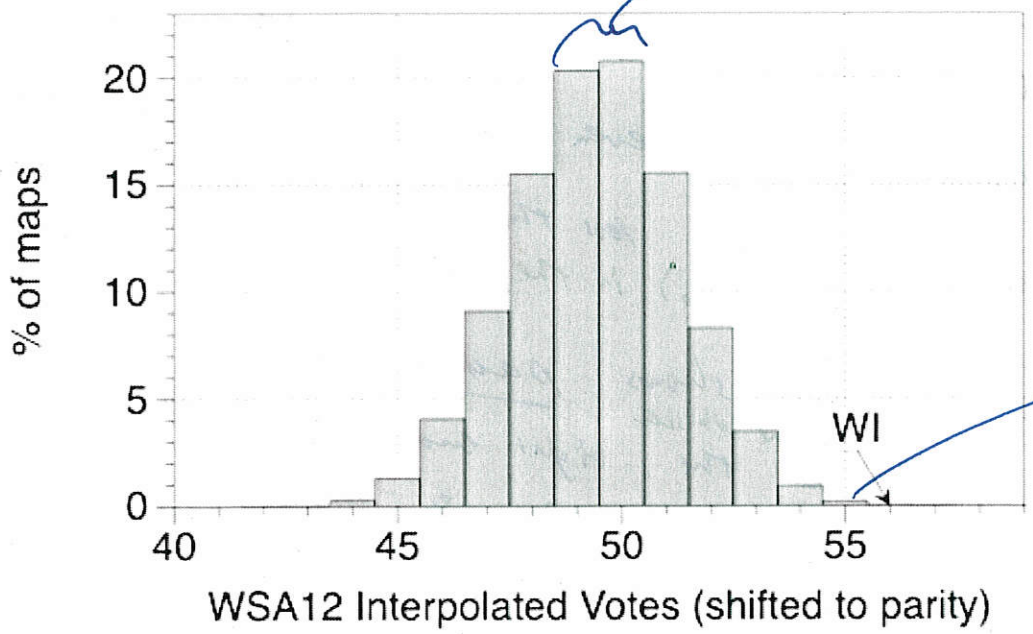
Is above threshhold $\Rightarrow$ gerrymandering

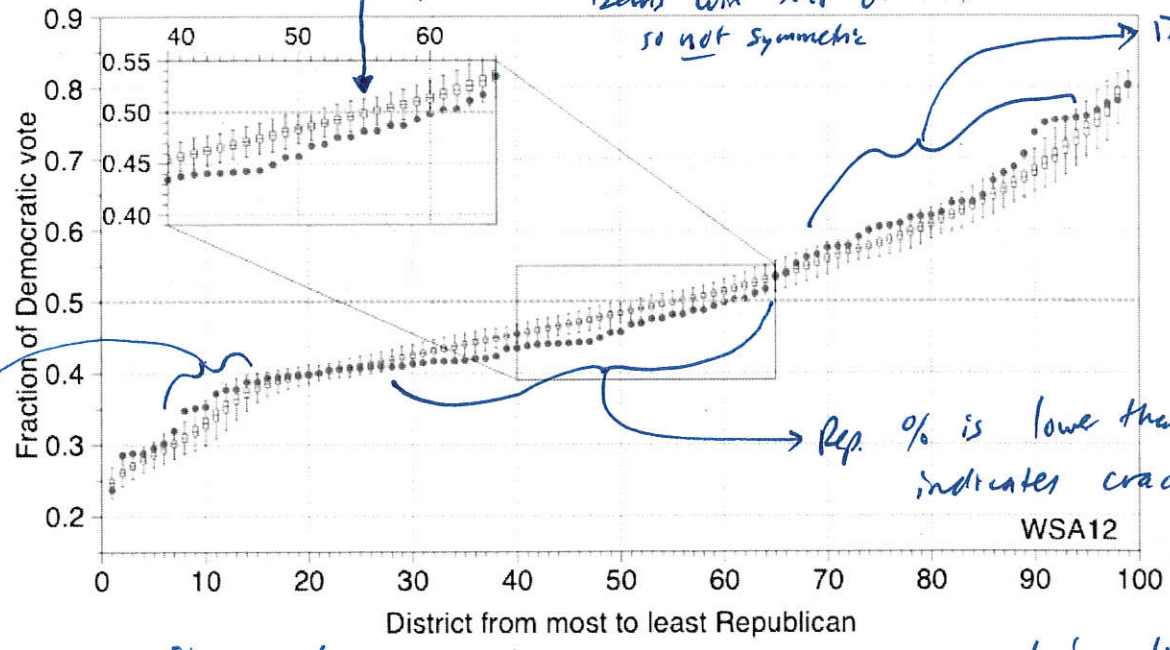
$8/9 \Rightarrow -0.253$

$7/9 \Rightarrow -0.142$

3. (5 points) Below are diagrams that represent an outlier analysis done for the State Assembly of Wisconsin in 2012. There are 99 districts in their State Assembly. In the bar graph (the top diagram), the blue bars represent the frequency that *Republicans* win a certain number of seats, and the "WI" is what the Wisconsin legislature's districting gave. In the bottom graph, there are box and whisker plots, and the red dots are the actual numbers for the Wisconsin legislature's districting. Note that a part of the diagram is enlarged to help you see what is happening more easily.

Here is the question: What does outlier analysis say about gerrymandering in Wisconsin, in terms of proportionality, packing and cracking, and/or partisan symmetry? Support your claims (as specifically as possible) with information from the diagrams. You can also use the next page for your answer.





49 or 50 is normal for Rep.

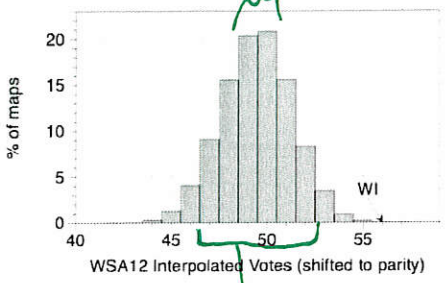
less than 1/2 of 1% would do 55, and WI has 56, big outlier

Normally 55 Reps (and 44 Dems) when Dems win 50% of vote, so not symmetric

Dem % is above normal, shows packing.

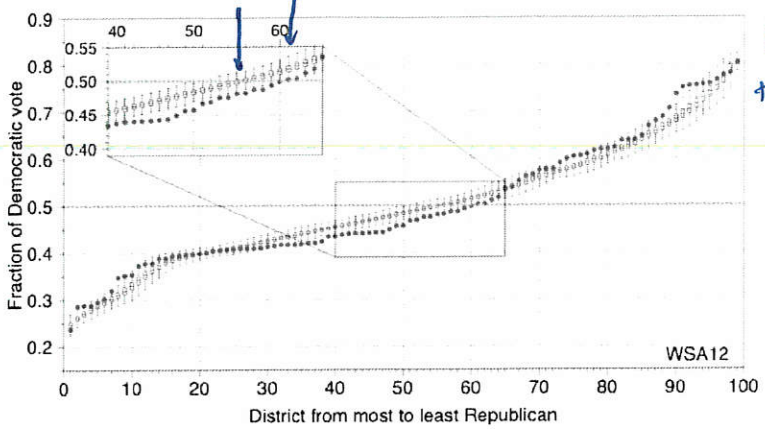
Rep. % is lower than normal indicates cracking

Rep. % is above normal, which shows they were trying to avoid packing Republicans.



Most of the time 49/50 Reps

90% of the time Reps win 53 or less seats.



# of seats Repr win with only 50% of the vote

This diagram shows that the WI map is an extreme outlier, electing more Republicans than even the most outlying elections in the bell curve (which are less than 10% of the possibilities.) Most of the time (about 90%) in the middle of the bell curve, Reps win 49 or 50 seats.

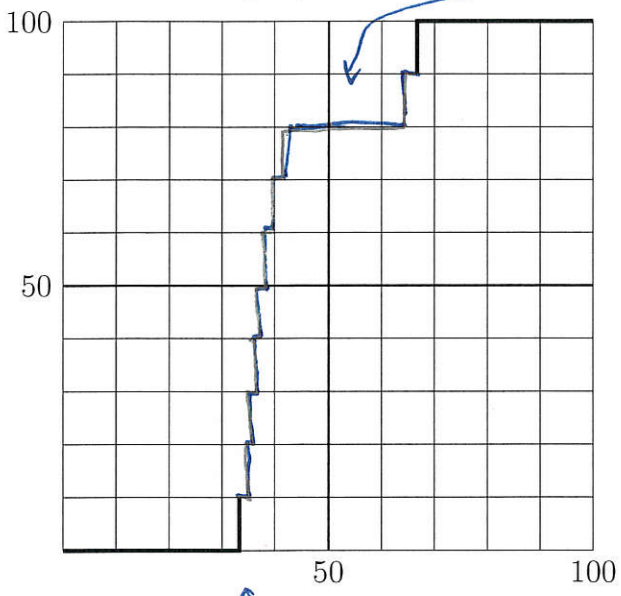
The other diagram shows packing by the Dem percentages being higher than the Whisker on the right end, and the Reps avoiding packing by making the Rep. % higher on the left end. In a "normal" districting, some Republican districts are packed by chance. The WI map has less of those than normal (below 50% and below the box/whisker plot in the enlarged area. This means Dems have not enough of the vote to win in those districts, and there are more of those than normal in the WI map.

The middle shows cracking as the Rep vote is lower

Can see than it is not nomally symmetric b/c when the box crosses 50% dem vote, Republicans still win 55 seats (55%), and that is normal. The actual map would give Republicans 60% of seats with only 50% of the vote (so not proportional either).

4. (5 points) Suppose a state has 10 districts, and when each party has 67% of the vote, they will win all of the districts (the graphs below show what that part of the symmetry graph would look like). Does that mean that the districting is fair and free of gerrymandering? Complete the symmetry graphs below to make examples that fit with the description above, *one* of which would indicate gerrymandering and the *other* that does not. Explain in words how the gerrymandered graph shows packing and/or cracking, *and* why the not gerrymandered graph demonstrates fairness.

IS gerrymandered:

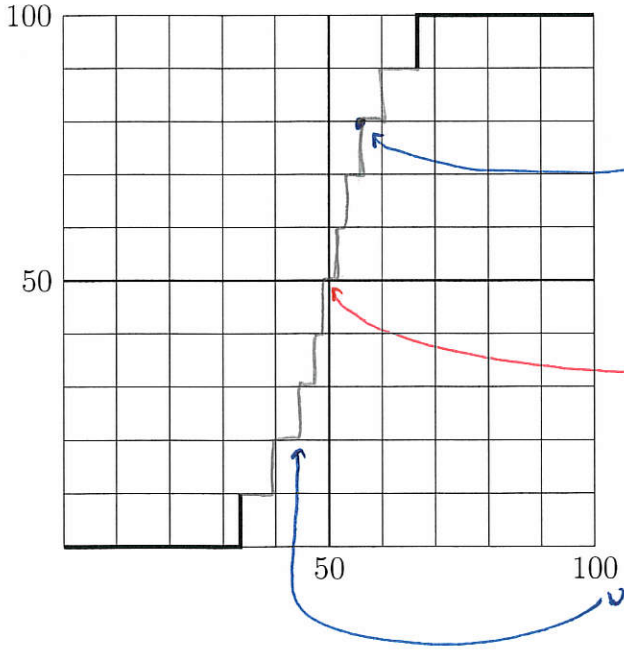


packing b/c it takes over 20% vote swing to flip the last 2 districts.

1.5

short steps shows cracking of the vote b/c there are a lot of districts that go for one party at the same time

Is NOT gerrymandered:



When the party wins 55% of the vote, they win 80% of the districts

1.5

When each party has 50% of vote, they each get 50% of representation.

when the party has 45% of the vote, other party has 55% and other party wins 80% (8/10) of seats. So it is symmetric and fair.

Extra Credit (1 point) Suppose a friend tells you the following: "My state is really unfairly gerrymandered! The redistricting made by the state legislature has low compactness scores. Like, the average Polsby-Popper score for the districts is a 0.27!"

What would you say to your friend about whether or not their state has been unfairly gerrymandered, based on what they have told you?

Lots of possible answers, but generally you cannot say for certain if it has been gerrymandered based on that 1 #.

- 0.27 *is* low, though. That does seem to show that something strange is going on with the drawing.
- You don't know *what* is going on, though. There may be lots of rivers / coastline that give a bad compactness score. You could also have strange district lines drawn in order to make the districts *more* symmetric, or proportional, or to ensure minority representation.
- More data is necessary:
  - → outlier analysis of compactness scores to see if 0.27 is "normal"
  - → other compactness scores
  - → partisan symmetry graph
  - → proportionality data
  - → efficiency gap scores.

Best is if give an example, like of two where you made districts less compact [illegible]

Formulas:

1 Compactness measures in Squaretopia

- **Skew measure:** W/L
- **Isoperimetric (Square Polsby-Popper) measure:** $16A/P^2$
- **Square Reock measure:** A/S

2 Real-world Compactness Measures

- **Harris:** W/L
- **Polsby-Popper:** $4\pi A/P^2$
- **Reock:** A/C

3 Efficiency Gap Formulas

- $EG = \dfrac{W_A - W_B}{\text{total votes}}$
- $EG = 2V - S - \dfrac{1}{2}$